OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS   **2641**

Probability & Statistics 1

Tuesday   **18 JANUARY 2005**   Afternoon   1 hour 20 minutes

Additional materials:
  Answer booklet
  Graph paper
  List of Formulae (MF8)

TIME   1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

*   Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
*   Answer **all** the questions.
*   Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
*   You are permitted to use a graphic calculator in this paper.

INFORMATION FOR CANDIDATES

*   The number of marks is given in brackets [ ] at the end of each question or part question.
*   The total number of marks for this paper is 60.
*   Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
*   **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 4 printed pages.

1   Siân shoots two arrows at a target. The probability that her first shot hits the target is 0.7. If her first shot hits the target then the probability that her second shot hits the target is 0.77. If her first shot misses the target then the probability that her second shot hits the target is 0.63. Find the probability that

   (i) Siân misses the target with both shots,   [2]

   (ii) Siân hits the target exactly once in her two shots.   [2]

2   A hockey coach has to choose a team of 11 players from a group of 11 men and 11 women.

   (i) If there is no restriction on the number of team members of each gender, find in how many different ways the coach can choose the team.   [2]

   (ii) If the team is chosen at random from the group, find the probability that it consists of 6 men and 5 women.   [4]

3   A company employs a large number of people in its city office and records show that 35% of the employees live outside the city limits. The Finance Department employs 18 men and 23 women. The number of these men who live outside the city limits is denoted by X and the number of these women who live outside the city limits is denoted by Y.

   (i) Assuming a binomial model, find $P(6 \leqslant X \leqslant 10)$.   [3]

   (ii) Assuming a binomial model, find $P(Y = 10)$.   [3]

   (iii) Give a reason why binomial models might not be suitable.   [1]

4   The table below shows the mean GCSE score and A Level mathematics grade of 6 randomly chosen students. The mean GCSE score for a student can take values from 0 to 8, with 8 representing the highest performance and 0 representing the lowest performance. The A Level grades are A, B, C, D, E and U, with A representing the highest performance and U the lowest performance.

Student	1	2	3	4	5	6
Mean GCSE score	4.1	5.2	6.4	5.3	7.4	3.6
A Level grade	E	C	D	B	A	U

   (i) Calculate Spearman's rank correlation coefficient for the data.   [4]

For the same 6 students Spearman's rank correlation coefficient between their GCSE mathematics grade and their A Level mathematics grade was 0.943. The head teacher of a school wishes to use **either** the mean GCSE score **or** the GCSE mathematics grade as a predictor of students' A Level mathematics grades.

   (ii) On the basis of the values of Spearman's rank correlation coefficient for these 6 students, state, giving a reason, which should be used.   [2]

   (iii) Give a reason why the choice of predictor might be wrong.   [1]

5   The owner of a car repair shop conducted a survey into the length of time that cars were in his repair shop undergoing repairs before being returned to their owners. He measured the time, t hours, that it took for each of a sample of 120 cars in his shop to be repaired and returned to their owners. The data are illustrated in the histogram below. 50 cars were repaired in the period $2 \leqslant t < 4$.
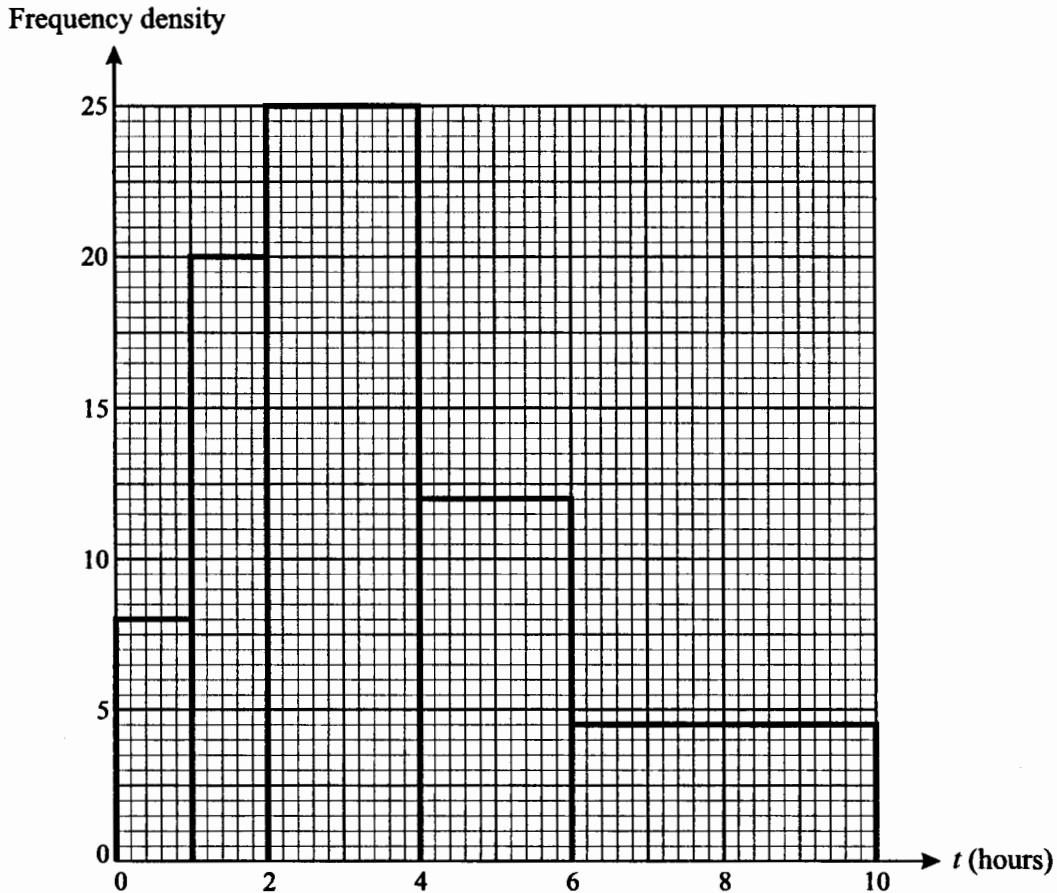
(i) Estimate the mean repair time for the sample of 120 cars.   [4]

(ii) Estimate the standard deviation of the repair times for the sample of 120 cars.   [3]

6   Alfie and Betty play rounds of a game by each tossing two unbiased coins. A round results in a 'matching' if **either** they both obtain two heads, **or** they both obtain two tails **or** they both obtain a head and a tail.

(i) Show that the probability that the first round results in a matching is $\frac{3}{8}$.   [3]

If the first round does not result in a matching, they continue to play rounds until a matching is obtained.

(ii) Find the probability that they play a total of 3 rounds or more.   [4]

(iii) Find the expectation of the total number of rounds played, and give its meaning in the context of the question.   [2]

7 In a science lesson, Shivani measured the masses of 20 small objects. Their masses, in grams to the nearest 0.1 g, are given below, in numerical order.

$$1.6 \quad 2.2 \quad 3.0 \quad 3.1 \quad 3.4 \quad 4.1 \quad 4.2 \quad 4.7 \quad 4.7 \quad 4.8$$
$$5.2 \quad 5.3 \quad 5.3 \quad 5.4 \quad 5.5 \quad 5.8 \quad 5.8 \quad 5.8 \quad 5.9 \quad 6.1$$

   (i) Find the median and interquartile range of this set of data. [3]

   (ii) Draw, on graph paper, a box-and-whisker plot of the data. [3]

During the same science lesson Emma also weighed 20 small objects, correct to the nearest 0.1 g, and obtained the following values:

     lower quartile 4.5 g,   median 5.1 g,   upper quartile 6.0 g.

   (iii) Compare Shivani's and Emma's data, referring both to the shapes of the distributions and to their variability. [3]

8 A teacher decided to investigate the connection between students' performances in History and Geography. She selected 5 students at random and recorded their scores, x and y, in History and Geography tests respectively. She found that, for this set of data, the regression line of y on x had equation $y = 18.5 + 0.1x$ and the regression line of x on y had equation $x = 16.6 + 0.4y$.

   (i) Using these equations, calculate the values of the mean score in the History test and the mean score in the Geography test for the 5 students. [3]

   (ii) Hence show that $\Sigma x = 125$ and $\Sigma y = 105$. [1]

It is given that $\Sigma x^2 = 3215$, $\Sigma y^2 = 2227.5$ and $\Sigma xy = 2634$.

   (iii) Obtain the product moment correlation coefficient for the data. [3]

   (iv) The Geography score of a sixth student was mislaid but his History score was known to be 26. Use one of the equations to estimate this student's Geography score, and give a reason for the use of the chosen equation. [3]

   (v) Comment on the reliability of your estimate. [1]

1. (i)	0.3×0.37	M1		Multiplying probs
	0.111	A1		
			2	
(ii)	$0.7 \times 0.23 + 0.3 \times 0.63$	M1		Both HM and MH
	0.35	A1		
			2	

2. (i)	$^{22}C_{11}$ seen or attempt with factorials	M1		
	705432	A1		
			2	
(ii)	$^{11}C_6$ and $^{11}C_5$ seen	M1		
	Multiplied	M1		
	$213444/705432$	M1		(Their $^{11}C_6 \times ^{11}C_5$)/their (i)
	$0.3025...$			
	$0.3025...$	A1		allow 2541k/8398k
			4	
	ALITER 2(ii)			
	$\dfrac{11}{6!5!} \times \left(\dfrac{11}{22} \times \dfrac{10}{21} \times \dfrac{9}{20} \times \dfrac{8}{19} \times \dfrac{7}{18} \times \dfrac{6}{17}\right) \times \left(\dfrac{11}{16} \times \dfrac{10}{15} \times \dfrac{9}{14} \times \dfrac{8}{13} \times \dfrac{7}{12}\right)$			
		M1		Both 5 and 6 term probabilities seen.
		M1		Both 5 and 6 term probabilities multiplied
		M1		Fully correct method including 11!/(6!5!)
		A1		0.303 or equiv.

3. (i)	Use B(18,0.35) table	M1		Or probs, at least one correct.
	$0.9788 - 0.3550$	M1		fully correct method
	$0.6238, 0.624$	A1		
			3	
(ii)	$^{22}C_{10}(0.35)^{10}(0.65)^{13}$	M1		allow p,q muddle
		M1		fully correct method
	$0.11668, 0.117$	A1		
			3	
(iii)	e.g. May be biased sample,			
	Residences may not be indep.	B1		any relevant reason
			1	

4. (i)	Student 1 2 3 4 5 6			
	Rank mean2 3 5 4 6 1			
	Rank grade 2 4 3 5 6 1	B1		correct ranks (or reversed)
	$\sum d^2 = 6$	M1		from ranked data.
	$r = 1 - (6 \times 6)/(6 \times (36-1))$	M1		fully correct method (their sum of d^2)
	$29/35$ or 0.829	A1		
			4	
(ii)	Use Mathematics grade	B1ft		
	Greater correlation	B1		
			2	
(iii)	e.g. Sample too small to generalise	B1		any relevant reason
			1	

5.(i)	t 0.5 1.5 3 5 8	B1		
	f 8 20 50 24 18	B1		
	mean=448/120	M1		from their f,t
	3.73(33....)	A1		
			4	

(ii)	Evaluate 2249/120	M1		from their f,t
	subtract mean^2 and sq.rt.	M1		
	2.19	A1		
			3	
	ALITER 5(ii)			
	e.g. $(0.5-3.733)^2$x8	M1		any correct term
				from their f,t
	Fully correct method	M1		from their f,t
	2.19	A1		
6.(i)	P(HH)matchings=$(1/4)^2$,			
	P(HT)matchings=$(1/2)^2$,			
	P(TT)matchings=$(1/4)^2$	B1		
	$(1/4)^2+(1/2)^2+(1/4)^2$	M1		allow even if errors in HT matchings
	3/8	A1		
	A.G.			
			3	
(ii)	Geometric distribution stated or implied.	M1		
	q^2 or 1-p-qp	M1		
	with q=5/8 (and p=3/8, if used)	M1		fully correct method
	25/64 or 0.391	A1		
			4	
(iii)	8/3 aef	B1		allow 2.67
	Refer to average number of matchings.	B1		
			2	
7(i)	median=(4.8+5.2)/2=5.0	B1		
	Find LQ(3.75),UQ(5.65)subtract	M1		allow slight variation
	1.9	A1ft		
			3	
(ii)	show 1.6,6.1,median and quartiles.	M1		recognisable attempt at box-plot
		A1ft		allow one error on diagram
		A1ft		fully correct (f.t.)
	NB Graph paper not used	M1(max)		
			3	
(iii)	Shivani's is positive skew			
	Emma's is negative skew	B1ft		for either
	Emma's IQR=1.5	B1		can be implied
	Shivani's is more variable	B1ft		
			3	
8(i)	Lines cross at mean x,mean y	M1		stated or implied
	e.g. y=18.5+0.1(16.6+0.4y)	M1		reasonable attempt at sim. equns.(to find both)
	mean y=21 , mean x=25	A1		
			3	
(ii)	5x25=125 and 5x21=105	B1		from correctly obtained means.
			1	
(iii)	$$\frac{9}{\sqrt{22.5\times 90}}$$	M1		any of Sxx,Sxy,Syy correct.
		M1		fully correct method
	0.2	A1		
			3	
(iv)	use y on x because x given, y needed.	B1		
	y=18.5+0.1x26	M1		
	y=21(.1)	A1		allow 2sf answer.
			3	
(v)	unreliable,r small	B1		any relevant reason
			1	